Chapter 2 Functions and Their Graphs

Course/Section
Lesson Number
Date

Section 2.1 Linear Equations in Two Variables

Section Objectives: Students will know how to find the slopes of lines and use slope to write and graph linear equations in two variables.

I. Using Slope (pp. 172-174) Pace: 10 minutes

- State that equations of the form $y = mx + b$ are called **linear equations in two variables.** They are called *linear* because their graphs are lines (i.e., straight lines).
- State that the **slope** of a line is a measure of its steepness.
- Consider the line given by the linear equation $y = mx + b$. By replacing x with zero, we see that the y-intercept of the line is $(0, b)$. Note that $(1, m + b)$ is also a point on the line. From this we can see that the slope of the line is m, since a one-unit change in x produces an m-unit change in y. State the following definition.

  The graph of an equation of the form $y = mx + b$ is a line with slope m and y-intercept $(0, b)$. This form is called **slope-intercept form**.
- Discuss the difference between lines with positive slope, negative slope, zero slope, and no slope. Draw a picture of each.

II. Finding the Slope of a Line (pp. 175-176) Pace: 10 minutes

- Define the slope of a line to be the ratio of the change in y to the change in x. In addition, if we know two points on the line, (x_1, y_1) and (x_2, y_2), then the change in y is $y_2 - y_1$ and the change in x is $x_2 - x_1$. Therefore, the slope m of a nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example 1. Find the slope of the line through each pair of points.

a) (3, -7) and (-4, 2)

$$m = \frac{2-(-7)}{-4-3} = -\frac{9}{7}$$

b) (2, -9) and (-6, -9)

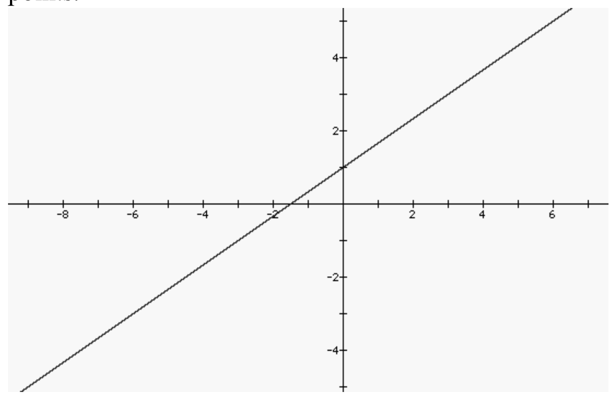
$$m = \frac{-9-(-9)}{-6-2} = \frac{0}{-8} = 0.$$ This line is horizontal.

c) (5, 4) and (5, -8)

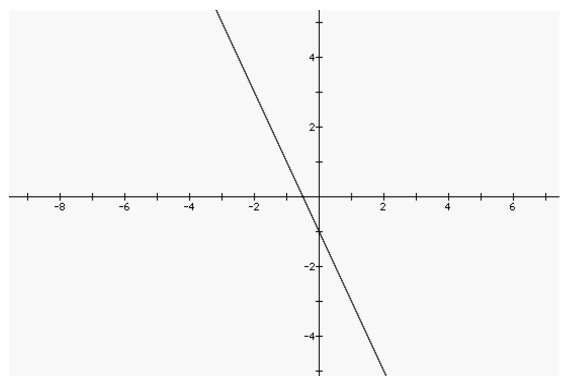
$$m = \frac{-8-4}{5-5} = \frac{-12}{0}$$, no slope

Example 2. Sketch the graph of the following.

a) $y = \frac{2}{3}x + 1$. Plot the y-intercept (0, 1). From this point go up 2 and to the right 3. This produces another point on the line. Now draw the line through these two points.



b) $y = -2x - 1$. Plot the y-intercept (0, -1). From this point go down 2 and to the right 1. This produces another point on the line. Now draw the line through these two points.



III. Writing
Linear Equations in Two Variables (pp. 177-178)
Pace: 15 minutes

- Discuss the following: If (x, y) is any other point on a line with slope m that contains (x_1, y_1), then

  $$\frac{y - y_1}{x - x_1} = m, \text{ or } y - y_1 = m(x - x_1)$$

  This is called the **point-slope form** of the equation of the line. State this definition as follows:
  Point-Slope Form: An equation of the line through the point (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$.

Example 3. Find the slope-intercept form of the equation of the line with slope 4 that passes through the point (-6, 2).

$$y - 2 = 4(x - (-6))$$
$$y - 2 = 4x + 24$$
$$y = 4x + 26$$

Tip: Inform the students that whenever they are instructed to find the equation of a line, they should think of point-slope form first.

Example 4. Find the slope-intercept form of the equation of the line that passes through the points (5, 1) and (-1, 3).

$$m = \frac{3-1}{-1-5} = \frac{2}{-6} = -\frac{1}{3}$$

$$y - 1 = -\frac{1}{3}(x-5)$$

$$y - 1 = -\frac{1}{3}x + \frac{5}{3}$$

$$y = -\frac{1}{3}x + \frac{8}{3}$$

- Note that the two forms of the equation of a line that we have so far depend on the slope. What do we do with vertical lines? State the following special forms of linear equations.
  1. An equation of the vertical line through any point with an x-coordinate of a is $x = a$.
  2. An equation of the horizontal line through any point with a y-coordinate of b is $y = b$.
  3. The general form of a linear equation is $Ax + By + C = 0$.

IV. Parallel and Perpendicular Lines (pp. 179-180) Pace: 10 minutes

- State the following two facts.
  1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is, $m_1 = m_2$.
  2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is, $m_1 = 1/m_2$.

Example 5. Find the general form of the equation of the line that passes through the point (1, -3) and is (a) parallel to and (b) perpendicular to the line given by $2x + 3y = 1$.
First find the slope of the given line by writing its equation in slope-intercept form.

$$2x + 3y = 1$$
$$3y = -2x + 1$$
$$y = (-2/3)x + 1/3$$

The slope of the given line is –2/3.

a) Parallel line:

$$y - (-3) = -2/3(x - 1)$$
$$3(y + 3) = -2(x - 1)$$
$$3y + 9 = -2x + 2$$
$$2x + 3y + 7 = 0$$

b) Perpendicular line:

$$y - (-3) = \frac{3}{2}(x-1)$$
$$2y + 6 = 3x - 3$$
$$3x - 2y - 9 = 0$$

V. Application (p. 180) Pace: 5 minutes

- If the cost of depreciation is the same amount every year, then we call this linear or straight-line depreciation.

Example 6. A company purchases a \$20,000 machine. In 4 years the machine will be worth \$10,000. Write a linear equation that relates the value *V* of the machine after *t* years.
First find the slope of the line through (0, 20,000) and (4, 10,000).

$$m = \frac{10,000 - 20,000}{4 - 0} = -\frac{10,000}{4} = -2500$$

$$V - 20,000 = -2500(t - 0)$$

$$V = -2500t + 20,000$$